

Relativistic Einstein-Podolsky-Rosen correlations for vector and tensor states

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We calculate and investigate the relativistic correlation function for bipartite systems of spin-1/2 in vector and spin-1 particles in tensor states. We show that the relativistic correlation function, which depends on particles momenta, may have local extrema. What is more, the momentum dependence of the correlation functions for two choices of relativistic spin operator may be significantly different.

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I. INTRODUCTION

We have recently shown that for the scalar states the relativistic Einstein-Podolsky-Rosen (EPR) correlation function, which depends on the particle four-momenta, may have local extrema for certain fixed configurations in massive particle systems [1–5]. Such extrema are present for bipartite systems of both spin-1/2 and spin-1 particles, for two different choices of the relativistic spin operator. This property has not been reported in any of the earlier papers on the subject [6–24].

Moreover we have shown that in certain configurations relativistic quantum correlations can be stronger than in the non-relativistic case, leading to stronger violation of the Bell-type inequalities.

The aim of the present paper is to extend our considerations on non-scalar states. We present and discuss new results for the correlation functions for vector states (for spin-1/2 particles) and antisymmetric tensor states (for spin-1 particles).

II. PRELIMINARIES

In order to calculate correlations between spin degrees of freedom, we need a properly defined relativistic spin operator. The difficulty is that there does not exist any unambiguous definition of such an operator [7–10, 12, 16, 17, 21–25]. In calculations of relativistic EPR correlation functions one most frequently uses the so called Newton-Wigner spin operator [26]

$$\hat{S}_{NW} = \frac{1}{m} \left(\hat{\mathbf{W}} - \hat{W}^0 \frac{\hat{\mathbf{P}}}{\hat{P}^0 + m} \right), \quad (1)$$

or the operator related to the projection of the center-of-mass (c.m.) spin operator on the direction $\boldsymbol{\omega}$ (see e.g. [9])

$$\hat{S}_{cm}(\boldsymbol{\omega}) = \frac{\boldsymbol{\omega} \cdot \hat{\mathbf{W}}}{\sqrt{m^2 + (\boldsymbol{\omega} \cdot \hat{\mathbf{P}})^2}}. \quad (2)$$

For the discussion of this point see [2]. Below we perform calculations for both choices of the spin operator.

The states, for which we calculate the relativistic correlation functions are defined as follows. The Hilbert space of a single particle of mass m and spin s is spanned by eigenvectors of the four-momentum operators \hat{P}^μ :

$$\hat{P}^\mu |q, \sigma\rangle = q^\mu |q, \sigma\rangle, \quad (3)$$

where σ is the value of the spin projection on the z axis; $\sigma = \{-s, -s+1, \dots, s\}$, and $q^\mu = (q^0, \mathbf{q})$. We assume the covariant normalization of the states (3).

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In EPR-type experiments one deals with bipartite states. For the sake of convenience we introduce the following notation for fermionic (half-integer s) and bosonic (integer s) bipartite states:

$$|(k, \sigma), (p, \lambda)\rangle_f = \frac{1}{\sqrt{2}}(|k, \sigma\rangle \otimes |p, \lambda\rangle - |p, \lambda\rangle \otimes |k, \sigma\rangle), \quad (4)$$

$$|(k, \sigma), (p, \lambda)\rangle_b = \frac{1}{\sqrt{2}}(|k, \sigma\rangle \otimes |p, \lambda\rangle + |p, \lambda\rangle \otimes |k, \sigma\rangle), \quad (5)$$

respectively.

Classification of all bipartite states covariant with respect to the Lorentz group action has been discussed in our earlier papers. For vector bosons it was done in Ref. [1] and for fermions in Ref. [8]. In particular for a bipartite system of spin-1/2 particles the vector state reads

$$|\varphi\rangle = -i\varphi^\mu (v^T(k)\gamma^2\gamma^0\gamma_\mu v(p))_{\sigma\lambda} |(k, \sigma), (p, \lambda)\rangle_f, \quad (6)$$

where γ_μ denotes the Dirac matrices (for the convention used see [8]), $|(k, \sigma), (p, \lambda)\rangle_f$ is defined by Eq. (4) and the Dirac field amplitude $v(q)$ ($q = \{k, p\}$) is a 2×4 matrix of the form [8]

$$v(q) = \frac{1}{2\sqrt{1 + \frac{q^0}{m}}} \begin{pmatrix} (\mathbb{1} + \frac{1}{m}q^\mu\sigma_\mu)\sigma_2 \\ (\mathbb{1} + \frac{1}{m}q^{\pi\mu}\sigma_\mu)\sigma_2 \end{pmatrix}, \quad (7)$$

where $q^\pi = (q^0, -\mathbf{q})$, $\sigma_\mu = (\mathbb{1}, \sigma_i)$ and σ_i , $i = \{1, 2, 3\}$ are the Pauli matrices.

The tensor state for a bipartite system of spin-1 particles has the form

$$|\Phi\rangle = T_{\mu\nu} e^\mu_\sigma(k) e^\nu_\lambda(p) |(k, \sigma), (p, \lambda)\rangle_b, \quad (8)$$

where $|(k, \sigma), (p, \lambda)\rangle_b$ is defined by Eq. (5) and the amplitude $e(q)$ ($q = \{k, p\}$) reads [1]

$$e(q) = \left(\frac{\frac{\mathbf{q}^T}{m}}{\mathbb{1} + \frac{\mathbf{q} \otimes \mathbf{q}^T}{m(m+q^0)}} \right) V^T, \quad V = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & i & 0 \end{pmatrix}. \quad (9)$$

As $T_{\mu\nu}$ we take the most general form of an antisymmetric tensor

$$T_{\mu\nu} = \begin{pmatrix} 0 & \alpha^j \\ -\alpha^i & \epsilon^{ijk}\beta^k \end{pmatrix}, \quad (10)$$

where α and β are arbitrary complex vectors fixing the polarization of the state (8).

III. CORRELATION FUNCTIONS

Now let us consider two distant observers in the same inertial frame of reference—Alice and Bob. Both share a pair of particles of mass m in one of the states (6) or (8). Alice measures the spin component of one particle along the direction \mathbf{a} and Bob measures the spin component of the other particle along the direction \mathbf{b} . Their observables are $\mathbf{a} \cdot \hat{\mathbf{S}}_{NW}$ and $\mathbf{b} \cdot \hat{\mathbf{S}}_{NW}$ for (1) and $\hat{S}_{cm}(\mathbf{a})$ and $\hat{S}_{cm}(\mathbf{b})$ for (2), respectively. The normalised correlation functions for an arbitrary bipartite state $|\psi\rangle$ are then given by

$$\mathcal{C}_{NW}^{\psi(k,p)}(\mathbf{a}, \mathbf{b}) = \frac{\langle \psi | (\mathbf{a} \cdot \hat{\mathbf{S}}_{NW})(\mathbf{b} \cdot \hat{\mathbf{S}}_{NW}) | \psi \rangle}{s^2 \langle \psi | \psi \rangle} \quad (11)$$

or

$$\mathcal{C}_{cm}^{\psi(k,p)}(\mathbf{a}, \mathbf{b}) = \frac{\langle \psi | \hat{S}_{cm}(\mathbf{a}) \hat{S}_{cm}(\mathbf{b}) | \psi \rangle}{s^2 \langle \psi | \psi \rangle}, \quad (12)$$

respectively.

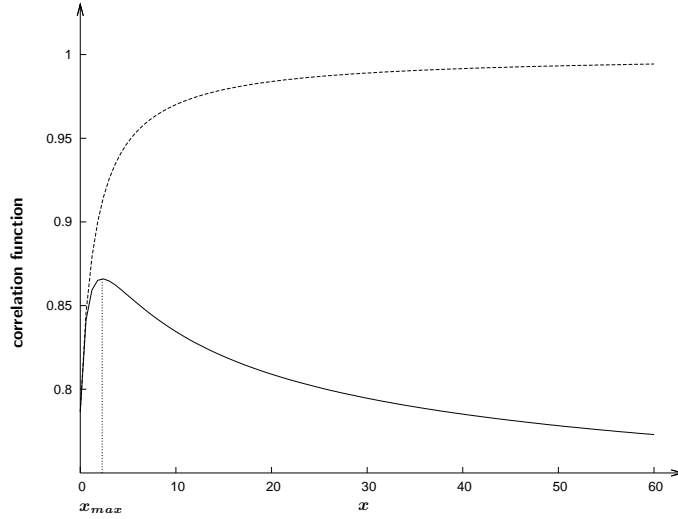


FIG. 1: The plot shows dependance of $C_{NW}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi})$ (solid line) and $C_{cm}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi})$ (dashed line) on x for $\mathbf{a} \cdot \mathbf{n} = 1$, $\mathbf{b} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{b} = 1/\sqrt{2}$, $\mathbf{a} \cdot \boldsymbol{\varphi} = \mathbf{n} \cdot \boldsymbol{\varphi} = 1/2$ and $\mathbf{b} \cdot \boldsymbol{\varphi} = -0.08$. The first one has maximum equal to 0.89 at $x = 2.30$ and the second one increases monotonically to 1.

A. The case of the system of spin-1/2 particles

In this subsection we calculate the correlation function for a spin-1/2 system in a vector state for the c.m. spin operator [c.f. Eq. (2)]. The corresponding correlation function for the Newton-Wigner spin operator [Eq. (1)] has been calculated elsewhere [8]. In view of lengthy formulae in a general case, we restrict our considerations to the case of the center-of-mass frame (c.m. frame), that is to a frame, in which the particles have opposite momenta ($\mathbf{p} = -\mathbf{k}$).

The correlation function for the Newton-Wigner spin operator (1) reads [8]

$$C_{NW}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi}) = \mathbf{a} \cdot \mathbf{b} - \frac{(x+1)[(\mathbf{a} \cdot \boldsymbol{\varphi})(\mathbf{b} \cdot \boldsymbol{\varphi}^*) + (\mathbf{a} \cdot \boldsymbol{\varphi}^*)(\mathbf{b} \cdot \boldsymbol{\varphi})]}{(x+1)|\boldsymbol{\varphi}|^2 - x|\boldsymbol{\varphi} \cdot \mathbf{n}|^2} - \frac{2x^2(\mathbf{a} \cdot \mathbf{n})(\mathbf{b} \cdot \mathbf{n})|\boldsymbol{\varphi} \cdot \mathbf{n}|^2}{(\sqrt{x+1}+1)^2[(x+1)|\boldsymbol{\varphi}|^2 - x|\boldsymbol{\varphi} \cdot \mathbf{n}|^2]} + \frac{x\sqrt{x+1}}{\sqrt{x+1}+1} \frac{(\mathbf{a} \cdot \mathbf{n})[(\mathbf{b} \cdot \boldsymbol{\varphi})(\mathbf{n} \cdot \boldsymbol{\varphi}^*) + (\mathbf{b} \cdot \boldsymbol{\varphi}^*)(\mathbf{n} \cdot \boldsymbol{\varphi})]}{(x+1)|\boldsymbol{\varphi}|^2 - x|\boldsymbol{\varphi} \cdot \mathbf{n}|^2} + \frac{x\sqrt{x+1}}{\sqrt{x+1}+1} \frac{(\mathbf{b} \cdot \mathbf{n})[(\mathbf{a} \cdot \boldsymbol{\varphi})(\mathbf{n} \cdot \boldsymbol{\varphi}^*) + (\mathbf{a} \cdot \boldsymbol{\varphi}^*)(\mathbf{n} \cdot \boldsymbol{\varphi})]}{(x+1)|\boldsymbol{\varphi}|^2 - x|\boldsymbol{\varphi} \cdot \mathbf{n}|^2}, \quad (13)$$

where $\mathbf{n} = \frac{\mathbf{k}}{|\mathbf{k}|}$, and $x = \left(\frac{|\mathbf{k}|}{m}\right)^2$. For c.m. operator (2) using Eqs. (12) and (6), we get

$$C_{cm}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi}) = \frac{\mathbf{a} \cdot \mathbf{b} + x(\mathbf{a} \cdot \mathbf{n})(\mathbf{b} \cdot \mathbf{n})}{\sqrt{1+x(\mathbf{a} \cdot \mathbf{n})^2}\sqrt{1+x(\mathbf{b} \cdot \mathbf{n})^2}} - \frac{(x+1)[(\mathbf{a} \cdot \boldsymbol{\varphi})(\mathbf{b} \cdot \boldsymbol{\varphi}^*) + (\mathbf{a} \cdot \boldsymbol{\varphi}^*)(\mathbf{b} \cdot \boldsymbol{\varphi})]}{\sqrt{1+x(\mathbf{a} \cdot \mathbf{n})^2}\sqrt{1+x(\mathbf{b} \cdot \mathbf{n})^2}[(x+1)|\boldsymbol{\varphi}|^2 - x|\boldsymbol{\varphi} \cdot \mathbf{n}|^2]}. \quad (14)$$

Both functions may have local extrema for specific values of arguments \mathbf{a} , \mathbf{b} , \mathbf{n} and $\boldsymbol{\varphi}$. Moreover, the dependence of the correlation functions on x differs for the spin operators (1) and (2). This is illustrated in Figs. 1–3, where the functions $C_{NW}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi})$ and $C_{cm}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi})$ of the argument x are drawn for three sets of parameters \mathbf{a} , \mathbf{b} , \mathbf{n} and $\boldsymbol{\varphi}$, as indicated in captions. In Fig. 1, the function $C_{NW}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi})$ has a local maximum, while $C_{cm}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi})$ monotonically approaches the value of 1 for $x \rightarrow \infty$. In Fig. 2 the function $C_{cm}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi})$ has a maximum, while $C_{NW}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi})$ decreases monotonically with increasing x . In Fig. 3 the function $C_{NW}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi})$ is constant and equal -0.5 , while $C_{cm}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi})$ increases monotonically with increasing x .

B. The case of the system of the spin-1 particles

The correlation function of a bipartite vector boson system in the state (8) can be calculated by means of Eqs. (8) and (11). For simplicity we give the result in the c.m. frame.

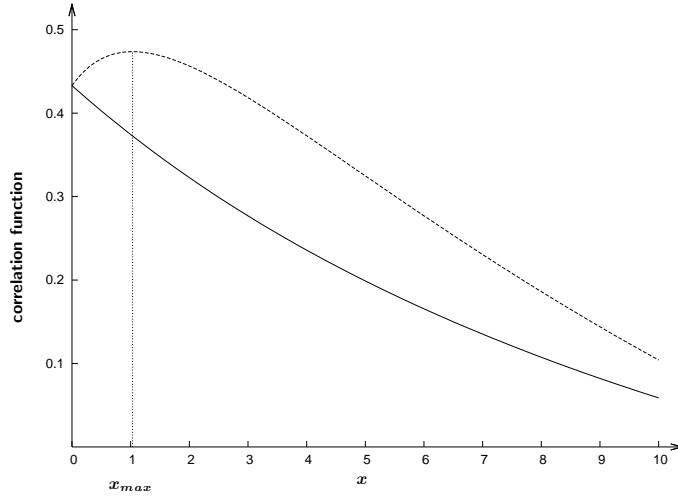


FIG. 2: The plot shows dependance of $C_{NW}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi})$ (solid line) and $C_{cm}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi})$ (dashed line) on x for $\mathbf{a} \cdot \mathbf{n} = 1$, $\mathbf{b} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{b} = -1/2$, $\mathbf{a} \cdot \boldsymbol{\varphi} = \mathbf{n} \cdot \boldsymbol{\varphi} = -0.97$ and $\mathbf{b} \cdot \boldsymbol{\varphi} = 0.48$. The first one decreases monotonically to -0.5 in the ultra-relativistic limit, the second one has maximum equal to 0.47 at $x = 1.03$ and its ultra-relativistic limit is -1 (full anticorrelation).

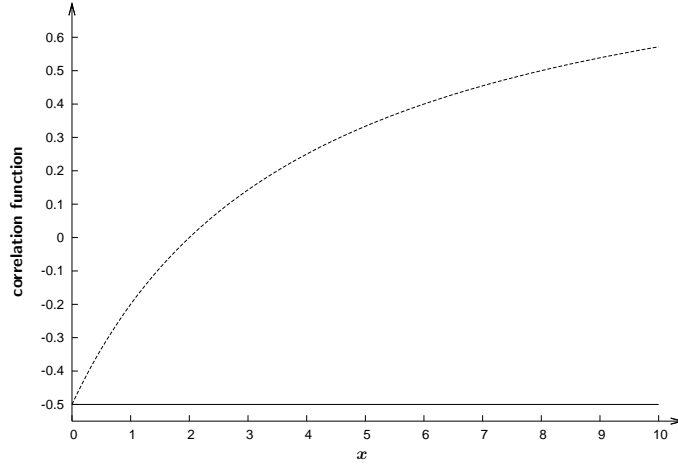


FIG. 3: The plot shows dependance of $C_{NW}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi})$ (solid line) and $C_{cm}^{\varphi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\varphi})$ (dashed line) on x for $\mathbf{a} \cdot \mathbf{n} = \mathbf{b} \cdot \mathbf{n} = 1/2$, $\mathbf{a} \cdot \mathbf{b} = 1/4$, $\mathbf{a} \cdot \boldsymbol{\varphi} = \mathbf{b} \cdot \boldsymbol{\varphi} = \sqrt{3}/2\sqrt{2}$ and $\mathbf{n} \cdot \boldsymbol{\varphi} = 0$. The first function is constant and equal -0.5 , the second one increases monotonically to reach 1 (full correlation) in the ultra-relativistic limit.

$$\begin{aligned}
C_{NW}^{\Phi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\alpha}, \boldsymbol{\beta}) = & \frac{1}{2[x(\boldsymbol{\beta} \cdot \mathbf{n})(\boldsymbol{\beta}^* \cdot \mathbf{n}) - x(2x+1)(\boldsymbol{\alpha} \cdot \mathbf{n})(\boldsymbol{\alpha}^* \cdot \mathbf{n}) - (x+1)(\boldsymbol{\beta} \cdot \boldsymbol{\beta}^*) - x(\boldsymbol{\alpha} \cdot \boldsymbol{\alpha}^*)]} \times \\
& \times \{ \sqrt{x}[(\boldsymbol{\alpha} \cdot \boldsymbol{\beta}^*)(\mathbf{n} \cdot (\mathbf{a} \times \mathbf{b})) + (\boldsymbol{\beta}^* \cdot \mathbf{n})(\boldsymbol{\alpha} \cdot (\mathbf{a} \times \mathbf{b}))] + (x+1)(\mathbf{a} \cdot \boldsymbol{\beta})(\mathbf{b} \cdot \boldsymbol{\beta}^*) \\
& - x(\mathbf{a} \cdot (\boldsymbol{\alpha}^* \times \mathbf{n}))(\mathbf{b} \cdot (\boldsymbol{\alpha} \times \mathbf{n})) + 2(\sqrt{x+1}-1)^2(\mathbf{a} \cdot \mathbf{n})(\mathbf{b} \cdot \mathbf{n})(\boldsymbol{\beta} \cdot \mathbf{n})(\boldsymbol{\beta}^* \cdot \mathbf{n}) \\
& + \sqrt{x}(\sqrt{x+1}-1)[2(\boldsymbol{\alpha} \cdot \mathbf{n})(\boldsymbol{\beta}^* \cdot \mathbf{n})(\mathbf{n} \cdot (\mathbf{a} \times \mathbf{b})) + (\mathbf{a} \cdot \mathbf{n})(\boldsymbol{\beta}^* \cdot \mathbf{n})(\mathbf{b} \cdot (\boldsymbol{\alpha} \times \mathbf{n}))] \\
& - \sqrt{x}(\sqrt{x+1}-1)[(\mathbf{a} \cdot \boldsymbol{\beta}^*)(\mathbf{b} \cdot (\boldsymbol{\alpha} \times \mathbf{n})) + (\mathbf{b} \cdot \mathbf{n})(\boldsymbol{\beta}^* \cdot \mathbf{n})(\mathbf{a} \cdot (\boldsymbol{\alpha} \times \mathbf{n})) - (\mathbf{b} \cdot \boldsymbol{\beta}^*)(\mathbf{a} \cdot (\boldsymbol{\alpha} \times \mathbf{n}))] \\
& - \sqrt{x+1}(\sqrt{x+1}-1)((\mathbf{a} \cdot \boldsymbol{\beta})(\mathbf{b} \cdot \mathbf{n})(\boldsymbol{\beta}^* \cdot \mathbf{n}) + (\mathbf{b} \cdot \boldsymbol{\beta})(\mathbf{a} \cdot \mathbf{n})(\boldsymbol{\beta}^* \cdot \mathbf{n})) \} + c.c.,
\end{aligned} \tag{15}$$

where again $\mathbf{n} = \frac{\mathbf{k}}{|\mathbf{k}|}$, $x = \left(\frac{|\mathbf{k}|}{m}\right)^2$, and $c.c.$ stands for the complex conjugation. For particular values of arguments, the function may have local extrema. In particular, choosing the polarization where $\boldsymbol{\alpha} = \mathbf{0}$, function (15) reduces to

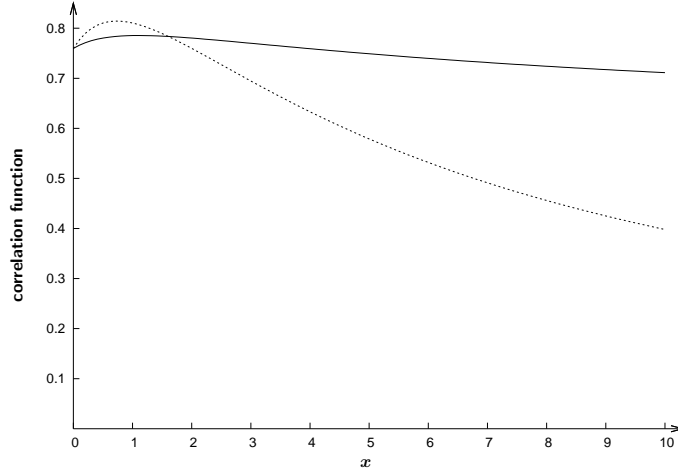


FIG. 4: The plot shows dependence of $C_{NW}^{\Phi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\beta})$ (solid line) and $C_{cm}^{\Phi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\beta})$ (dashed line) for configuration $\mathbf{a} \cdot \mathbf{n} = -\mathbf{b} \cdot \mathbf{n} = -1/2$, $\mathbf{a} \cdot \mathbf{b} = -0.90$, $\mathbf{a} \cdot \boldsymbol{\beta} = -0.79$, $\mathbf{b} \cdot \boldsymbol{\beta} = 0.97$ and $\boldsymbol{\beta} \cdot \mathbf{n} = 1/\sqrt{2}$. Both functions have local maxima — the first one equal to 0,79 for $x = 0.81$, the second one 1.10 for $x = 0.73$. Their ultra-relativistic limits are equal $3/4\sqrt{2}$ and 0 respectively.

the simpler form

$$C_{NW}^{\Phi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\beta}) = \frac{1}{x|\boldsymbol{\beta} \cdot \mathbf{n}|^2 - x - 1} \left\{ (x+1)(\mathbf{a} \cdot \boldsymbol{\beta})(\mathbf{b} \cdot \boldsymbol{\beta}) + (\sqrt{x+1}-1)^2(\mathbf{a} \cdot \mathbf{n})(\mathbf{b} \cdot \mathbf{n})(\boldsymbol{\beta} \cdot \mathbf{n})^2 \right. \\ \left. - (x+1-\sqrt{x+1})(\boldsymbol{\beta} \cdot \mathbf{n})[(\mathbf{a} \cdot \mathbf{n})(\mathbf{b} \cdot \boldsymbol{\beta}) + (\mathbf{a} \cdot \boldsymbol{\beta})(\mathbf{b} \cdot \mathbf{n})] \right\}. \quad (16)$$

Now taking into account that for any particle carrying four-momentum \mathbf{q}

$$\hat{S}(\boldsymbol{\omega}) = \frac{1}{\sqrt{m^2 + (\boldsymbol{\omega} \cdot \mathbf{q})^2}} \left[m(\boldsymbol{\omega} \cdot \hat{\mathbf{S}}_{NW}) + \hat{W}^0 \frac{\boldsymbol{\omega} \cdot \mathbf{q}}{m + q^0} \right], \quad (17)$$

$\boldsymbol{\omega} = \{\mathbf{a}, \mathbf{b}\}$, for the spin operator (2), we have

$$C_{cm}^{\Phi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\beta}) = \frac{(x+1)(\mathbf{a} \cdot \boldsymbol{\beta})(\mathbf{b} \cdot \boldsymbol{\beta})}{\sqrt{1+x(\mathbf{a} \cdot \mathbf{n})^2} \sqrt{1+x(\mathbf{b} \cdot \mathbf{n})^2} (x|\boldsymbol{\beta} \cdot \mathbf{n}|^2 - x - 1)}. \quad (18)$$

As in the case of spin-1/2 particles, the dependence of the correlation functions corresponding to the spin operators (1) and (2) may have significantly different dependences on the variable x . This is demonstrated in Figs. 4–7. In Fig. 4 both functions have local maxima. In Figs. 5 and 6 we have cases, for which one of the functions is constant, while the other one is a monotonic function of x . In Fig. 7 the function $C_{NW}^{\Phi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\beta})$ is monotonically increasing to reach the value 0.75 in the ultra-relativistic limit, while the function $C_{cm}^{\Phi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\beta})$ monotonically decreases to reach 0 in infinity.

IV. CONCLUSIONS

In our previous papers [1, 2], we have shown that the relativistic correlation functions in bipartite systems of spin-1/2 and spin-1 particles in a singlet state may have local extrema in terms of particles momentum for particular values of arguments. In this paper we have shown that a similar property appears for the correlation functions in states which are not Lorentz singlets.

The correlation functions have been calculated for two different choices of the relativistic spin operator, and in both cases the functions showed qualitatively similar behaviour. Nevertheless for particular values of parameters, the behaviour of the correlation functions for different spin operators could be quantitatively different. This feature could be used for testing which relativistic spin operator gives predictions which are in better agreement with the experimental data.

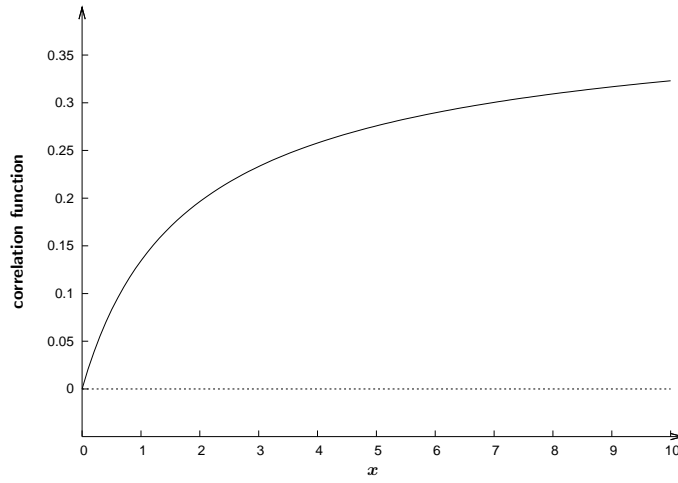


FIG. 5: The plot shows dependence of $C_{NW}^{\Phi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\beta})$ (solid line) and $C_{cm}^{\Phi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\beta})$ (dashed line) for configuration $\mathbf{a} \cdot \mathbf{n} = \boldsymbol{\beta} \cdot \mathbf{n} = 1/2$, $\mathbf{b} \cdot \mathbf{n} = \sqrt{3}/2$, $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \boldsymbol{\beta} = 0$ i $\mathbf{a} \cdot \boldsymbol{\beta} = 1$. The function for (1) increases monotonically from 0 to $\sqrt{3}/4$ in the ultra-relativistic limit.

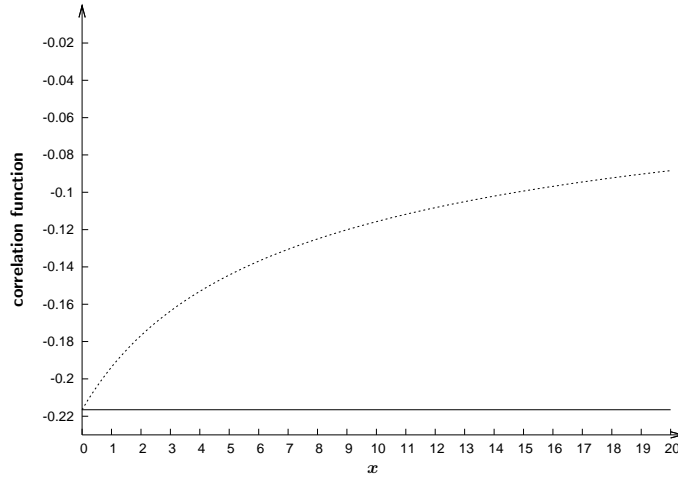


FIG. 6: The plot shows dependence of $C_{NW}^{\Phi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\beta})$ (solid line) and $C_{cm}^{\Phi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\beta})$ (dashed line) for configuration $\mathbf{a} \cdot \mathbf{n} = -\mathbf{b} \cdot \boldsymbol{\beta} = 1/2$, $\mathbf{b} \cdot \mathbf{n} = \boldsymbol{\beta} \cdot \mathbf{n} = 0$, $\mathbf{a} \cdot \mathbf{b} = \sqrt{3}/2$ i $\mathbf{a} \cdot \boldsymbol{\beta} = -\sqrt{3}/4$. The function for (1) is constant and equal to $-\sqrt{3}/8$, function for (2) monotonically increases from $-\sqrt{3}/8$ to 0 in the ultra-relativistic limit.

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- [1] P. Caban, J. Rembieliński, and M. Włodarczyk, Phys. Rev. A **77**, 012103 (2008).
 - [2] P. Caban, J. Rembieliński, and M. Włodarczyk, Phys. Rev. A **79**, 014102 (2009).
 - [3] P. Caban, Phys. Rev. A **77**, 062101 (2008).
 - [4] P. Caban, A. Dziągiewska, A. Karmazyn, and M. Okrasa, Phys. Rev. A **81**, 032112 (2010).
 - [5] J. Rembieliński and K. A. Smoliński, EPL **88**, 10005 (2009).
 - [6] D. Ahn, H. J. Lee, Y. H. Moon, and S. W. Hwang, Phys. Rev. A **67**, 012103 (2003).
 - [7] P. Caban and J. Rembieliński, Phys. Rev. A **72**, 012103 (2005).

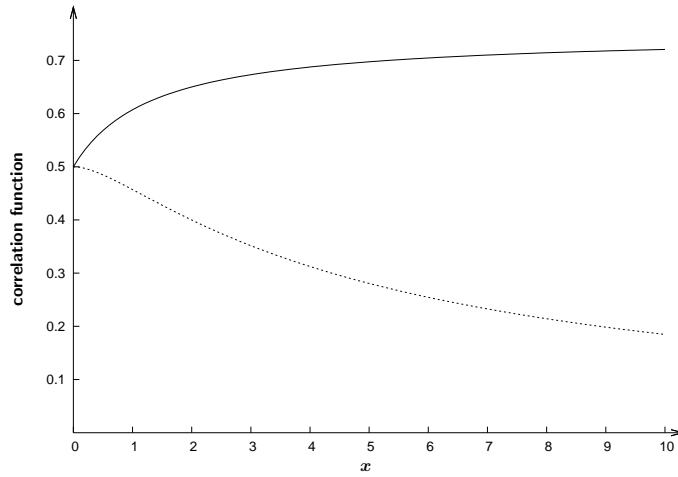


FIG. 7: The plot shows dependence of $C_{NW}^{\Phi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\beta})$ (solid line) and $C_{cm}^{\Phi(k,k^\pi)}(x, \mathbf{a}, \mathbf{b}, \boldsymbol{\beta})$ (dashed line) for configuration $\mathbf{a} \cdot \mathbf{n} = \mathbf{b} \cdot \mathbf{n} = \boldsymbol{\beta} \cdot \mathbf{n} = 1/2$, $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \boldsymbol{\beta} = -1/2$ i $\mathbf{a} \cdot \boldsymbol{\beta} = 1$. For (2), the correlations fade away with the increase of particles momenta and for (1) the particles become more correlated. in the ultra-relativistic limit correlation function for (1) operator equals 0.75.

- [8] P. Caban and J. Rembieliński, Phys. Rev. A **74**, 042103 (2006).
- [9] M. Czachor, Phys. Rev. A **55**, 72 (1997).
- [10] M. Czachor and M. Wilczewski, Phys. Rev. A **68**, 010302(R) (2003).
- [11] M. Czachor, Phys. Rev. Lett. **94**, 078901 (2005).
- [12] M. Czachor, Quantum Information Processing **9**, 171 (2010).
- [13] N. Friis, R. A. Bertlmann, M. Huber, and B. Hiesmayr, Phys. Rev. A **81**, 042114 (2010).
- [14] R. M. Gingrich and C. Adami, Phys. Rev. Lett. **89**, 270402 (2002).
- [15] R. M. Gingrich, A. J. Bergou, and C. Adami, Phys. Rev. A **68**, 042102 (2003).
- [16] H. Li and J. Du, Phys. Rev. A **68**, 022108 (2003).
- [17] D. Lee and E. Chang-Young, New Journal of Physics **6**, 67 (2004).
- [18] W. T. Kim and E. J. Son, Phys. Rev. A **71**, 014102 (2005).
- [19] A. Peres and D. R. Terno, Rev. Mod. Phys. **76**, 93 (2004).
- [20] A. Peres, P. F. Scudo, and D. R. Terno, Phys. Rev. Lett. **94**, 078902 (2005).
- [21] J. Rembieliński and K. A. Smoliński, Phys. Rev. A **66**, 052114 (2002).
- [22] C. Soo and C. C. Y. Lin, Int. J. Quantum Information **2**, 183 (2004).
- [23] H. Terashima and M. Ueda, Int. J. Quantum Information **1**, 93 (2003).
- [24] D. R. Terno, Phys. Rev. A **67**, 014102 (2003).
- [25] N. N. Bogolubov, A. A. Logunov, and I. T. Todorov, *Introduction to Axiomatic Quantum Field Theory* (Benjamin, Reading, MA, 1975).
- [26] T. D. Newton and E. P. Wigner, Rev. Mod. Phys. **21**, 400 (1949).